Covariant Image Reconstruction

Further development of the mathematical tools behind the Adobe® Photoshop® Healing Brush

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Poisson Cloning

Covariant Cloning

Adaption of Human Vision

Poisson Equation

\[ \Delta f(x, y) = \Delta g(x, y) \]

\[ \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \]

\[ \begin{align*}
\frac{\partial}{\partial x} & \rightarrow \frac{\partial}{\partial x} + A_1(x, y) \\
\frac{\partial}{\partial y} & \rightarrow \frac{\partial}{\partial y} + A_2(x, y)
\end{align*} \]

Covariant Derivative

\[ \frac{\partial}{\partial x} \frac{\partial}{\partial y} f + \frac{\partial}{\partial y} \frac{\partial}{\partial x} f = 0, \]

\[ (\frac{\partial}{\partial x} + A_1)(\frac{\partial}{\partial y} + A_1)f + (\frac{\partial}{\partial y} + A_2)(\frac{\partial}{\partial x} + A_2)f = 0. \]

Covariant Laplace Equation

\[ \Delta f + f \text{div} \mathbf{A} + 2 \mathbf{A} \cdot \text{grad} f + \mathbf{A} \cdot \mathbf{A} f = 0. \]

\[ \begin{align*}
\frac{\partial}{\partial x} g(x, y) & = 0 \\
\frac{\partial}{\partial y} g(x, y) & = 0 \\
\mathbf{A}(x, y) & = \frac{\text{grad} g}{g}
\end{align*} \]

Covariant Image Reconstruction

\[ \frac{\Delta f}{f} - 2 \frac{\text{grad} f}{f} \frac{\text{grad} g}{g} \frac{\Delta g}{g} + \frac{1}{2} (\text{grad} g) \cdot (\text{grad} g) = 0. \]