Covariant Image Reconstruction

Further development of the mathematical tools behind the Adobe® Photoshop® Healing Brush

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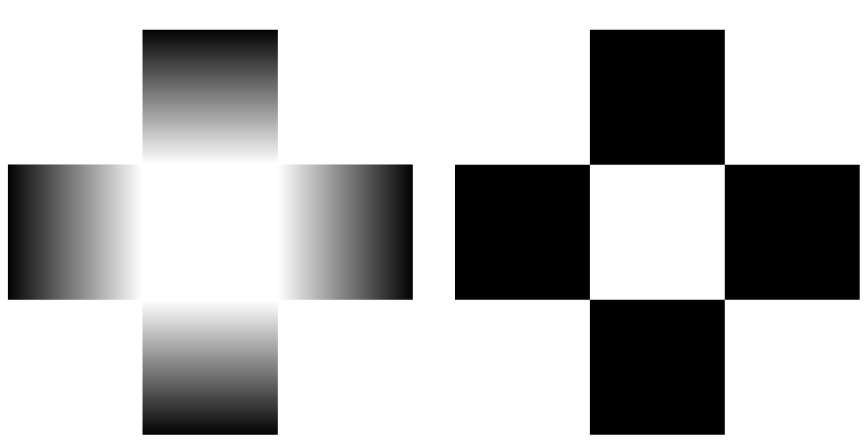
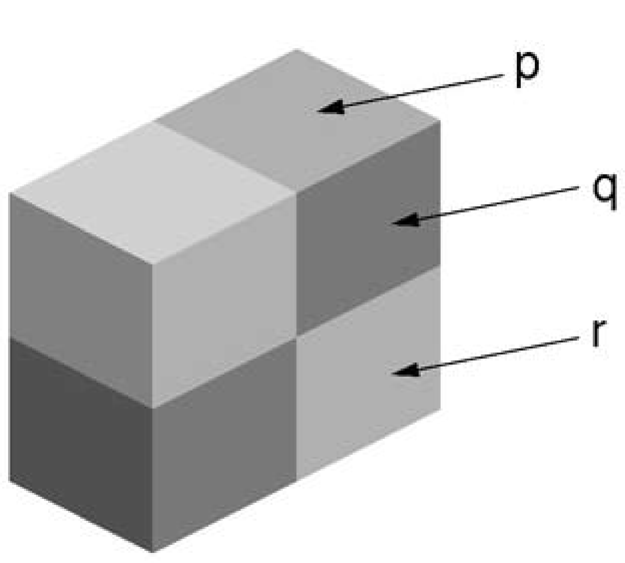
Poisson Cloning

Covariant Cloning

Adaption of Human Vision





Poisson Equation

$$\triangle f(x,y) = \triangle g(x,y)$$

$$\triangle = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}.$$

Covariant Derivative

$$\frac{\partial}{\partial x} \longrightarrow \frac{\partial}{\partial x} + A_1(x,y)$$

$$\frac{\partial}{\partial y} \longrightarrow \frac{\partial}{\partial y} + A_2(x,y)$$

$$\frac{\partial}{\partial x}\frac{\partial}{\partial x} f + \frac{\partial}{\partial y}\frac{\partial}{\partial y} f = 0,$$

$$\left(\frac{\partial}{\partial x} + A_1\right)\left(\frac{\partial}{\partial x} + A_1\right) f + \left(\frac{\partial}{\partial y} + A_2\right)\left(\frac{\partial}{\partial y} + A_2\right) f = 0,$$

Covariant Laplace Equation

$$\triangle f + f\, div\mathbf{A} + 2\mathbf{A}\cdot grad f + \mathbf{A}\cdot\mathbf{A} f = 0.$$

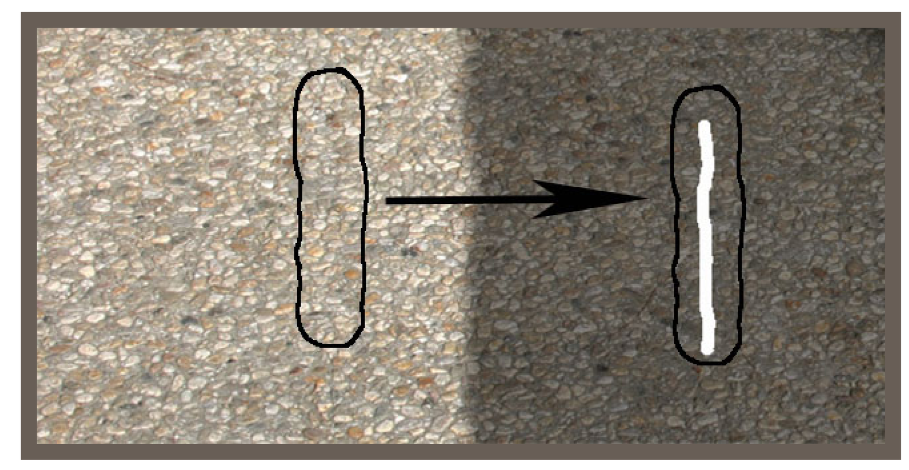
$$\left(\frac{\partial}{\partial x} + A_1(x,y)\right) g(x,y) = 0$$

$$\left(\frac{\partial}{\partial y} + A_2(x,y)\right) g(x,y) = 0$$

$$\mathbf{A}(x,y) = -\frac{grad g}{g}$$

Covariant Image Reconstruction

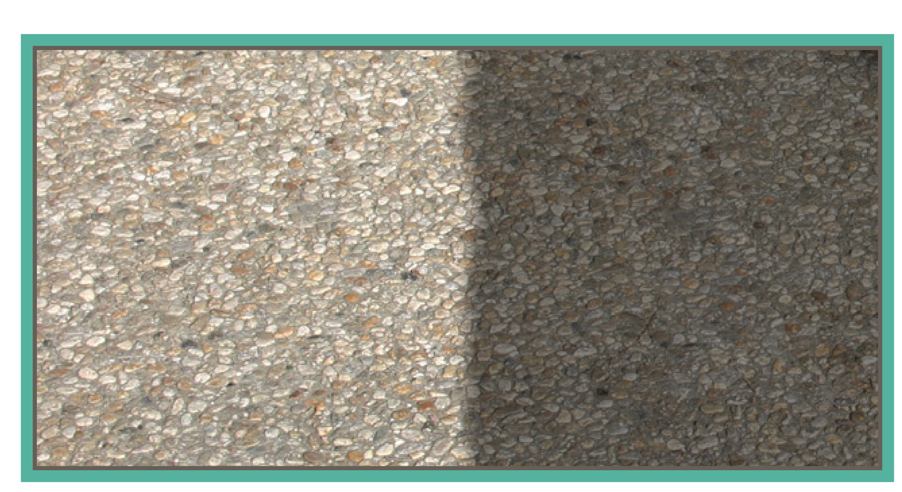
$$\frac{\triangle f}{f} - 2\frac{grad f}{f}\cdot\frac{grad g}{g} - \frac{\triangle g}{g} + 2\frac{(grad g)\cdot(grad g)}{g^2} = 0.$$



Orginal image of pebbles and a scratch. Source area used for Poisson cloing and covariant reconstruction.

Scratch removed by simple inpainting.

Scratch removed by Poisson cloning from the illuminated area.



Scratch removed by covariant cloning from the illuminated area as in above Figure.